

Quantum Mechanics Equation sheet

$$\Phi(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \Psi(x, t) dx$$

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$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$

$$[AB, C] = A[B, C] + [A, C]B$$

$$[f(x), p] = i\hbar \frac{df}{dx}$$

$$-\frac{\hbar}{2m} \nabla^2 \psi + V\psi = E\psi$$

$$P_l^m(x) \equiv (1-x^2)^{|m|/2} \left(\frac{d}{dx} \right)^{|m|} P_l(x)$$

$$P_l(x) \equiv \frac{1}{2^l l!} \left(\frac{d}{dx} \right)^l (x^2-1)^l$$

$$Y_l^m(\theta, \phi) = \epsilon \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}} e^{im\phi} P_l^m(\cos\theta)$$

$$\epsilon = (-1)^m \text{ for } m \geq 0 \text{ and } \epsilon = 1 \text{ for } m \leq 0$$

$$u(r) \equiv rR(r)$$

$$-\frac{\hbar^2}{2m} \frac{d^2 u}{dr^2} + \left[V + \frac{\hbar^2}{2m} \frac{l(l+1)}{r} \right] u = Eu$$

$$u(r) = Ar j_l(kr) + Br n_l(kr)$$

$$j_l(x) \equiv (-x)^l \left(\frac{1}{x} \frac{d}{dx} \right)^l \frac{\sin x}{x}$$

$$n_l(x) \equiv -(-x)^l \left(\frac{1}{x} \frac{d}{dx} \right)^l \frac{\cos x}{x}$$

$$E_{nl} = \frac{\hbar^2}{2ma^2} \beta_{nl}^2$$

$$E_n = - \left[\frac{m}{2\hbar^2} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \right] \frac{1}{n^2} = \frac{E_1}{n^2}$$

$$a \equiv \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.529 \times 10^{-10} \text{m}$$

$$\psi_{nlm} = \sqrt{\left(\frac{2}{na} \right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}} e^{-r/na} \left(\frac{2r}{na} \right)^l \times [L_{n-l-1}^{2l+1}(2r/na)] Y_l^m(\theta, \phi)$$

$$[L_x, L_y] = i\hbar L_z; \quad [L_y, L_z] = i\hbar L_x; \quad [L_z, L_x] = i\hbar L_y$$

$$L_{\pm} \equiv L_x \pm iL_y$$

$$L^2 f_l^m = \hbar^2 l(l+1) f_l^m; \quad L_z f_l^m = \hbar m f_l^m$$

$$L_{\pm} f_l^m = (A_l^m) f_l^{m\pm 1}$$

$$A_l^m = \hbar \sqrt{l(l+1) - m(m\pm 1)} = \hbar \sqrt{(l \mp m)(l \pm m + 1)}$$

$$S^2 |s m\rangle = \hbar^2 s(s+1) |s m\rangle; \quad S_z |s m\rangle = \hbar m |s m\rangle$$

$$\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

$$\sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$s = (s_1 + s_2), (s_1 + s_2 - 1), (s_1 + s_2 - 2), \dots, |s_1 - s_2|$$

$$|s m\rangle = \sum_{m_1+m_2=m} C_{m_1 m_2 m}^{s_1 s_2 s} |s_1 m_1\rangle |s_2 m_2\rangle$$

$$|s_1 m_1\rangle |s_2 m_2\rangle = \sum_s C_{m_1 m_2 m}^{s_1 s_2 s} |s m\rangle$$

$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle$$

$$\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{(E_n^0 - E_m^0)} \psi_m^0$$

$$E_n^2 = \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{(E_n^0 - E_m^0)}$$

$$E_{\pm}^1 = \frac{1}{2} [W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}]$$

$$T = \sqrt{p^2 c^2 + m^2 c^4} - mc^2$$

$$\psi(x) \cong \frac{C}{\sqrt{p(x)}} e^{\pm \frac{i}{\hbar} \int p(x) dx}$$