

Homework for October 16

I will be gone to a conference Oct 9-13 so no class that week. However, you should find a buddy and complete the following assignment as a group project (group size=2). Or, if you prefer, you may complete it on your own.

- (1) Prove that the ISBN check digit catches all single digit errors, and transposition errors.
- (2) Does the ISBN check digit catch all 2 digit errors? (2 digits entered incorrectly but not necessarily next to each other.)
- (3) Use the Euclidean algorithm to compute $\gcd(F_4, F_5)$, $\gcd(F_5, F_6)$, $\gcd(F_6, F_7)$ and $\gcd(F_7, F_8)$ where F_n is the n th Fibonacci number.
 - (a) Make a conjecture about $\gcd(F_n, F_{n+1})$.
 - (b) Prove your conjecture is valid.
 - (c) Find a formula for the number of steps in the Euclidean algorithm to compute $\gcd(F_n, F_{n+1})$. Your formula should be in terms of n .
- (4) For which n is $F_n \equiv 0 \pmod{2}$? Hint: $\equiv 0 \pmod{2}$ just means a number is even. Find a formula for all n that will satisfy this condition. Prove that your formula works. Induction will probably help.
- (5) For which n is $F_n \equiv 0 \pmod{3}$? Find a formula for all n that will satisfy this condition.
- (6) For which n is $F_n \equiv 0 \pmod{5}$? Find a formula for all n that will satisfy this condition.
- (7) Compute $\gcd(F_3, F_6)$, $\gcd(F_3, F_9)$, $\gcd(F_3, F_{12})$. Make a conjecture. How is this conjecture related to a previous conjecture?
- (8) Compute $\gcd(F_4, F_8)$, $\gcd(F_4, F_{12})$, $\gcd(F_4, F_{16})$. Make a conjecture. How is this conjecture related to a previous conjecture?
- (9) Generalize the last two conjectures. For example, compute $\gcd(F_5, F_{10})$, $\gcd(F_6, F_{12})$, $\gcd(F_7, F_{21})$
- (10) Fibonacci and modular arithmetic
 - (a) Examine the entire F_n sequence mod 2 (see maple commands below) and explain what pattern you see.
 - (b) Repeat the above step mod 3.
 - (c) Examine the entire F_n sequence mod 4 and explain what pattern you see.
 - (d) Examine the entire F_n sequence mod 5 and explain what pattern you see. Hint: dude, this pattern is far out! (that is, you'll need to go far out in the sequence to see this pattern.)

Useful maple commands:

with(combinat, fibonacci): (just enter this command once at the beginning of a session, it calls up a fibonacci package. **None of the other commands will work if you don't put this one in first.**)

fibonacci(34); (this returns the 34th fibonacci number)

seq(fibonacci(k) mod 2, k=1..20); (this reduces mod 2 and prints the first 20 terms in the fibonacci sequence.)

Note: excel will also do mod n. The command there is =mod(5,3). this command reduces 5 mod 3. You can use a fill down command to generate the fibonacci sequence.

Note: you are welcome to use other software if you like.