HW 26 Ch 27 Pr.2, 10, 20

2. In a Young’s double-slit experiment, the seventh dark fringe is located 0.025 m to the side of the central bright fringe on a flat screen, which is 1.1 m away from the slits. The separation between the slits is $1.4 \times 10^{-4}$ m. What is the wavelength of the light being used?

**REASONING** According to Equation 27.2, the wavelength $\lambda$ of the light is related to the angle $\theta$ between the central bright fringe and the seventh dark fringe according to

$$\lambda = \frac{d \sin \theta}{m + \frac{1}{2}}$$

where $d$ is the separation between the slits, and $m = 0, 1, 2, 3, \ldots$ The first dark fringe occurs when $m = 0$, so the seventh dark fringe occurs when $m = 6$. The distance $d$ is given, and we can determine the angle $\theta$ by using the inverse tangent function, $\theta = \tan^{-1} \left( \frac{y}{L} \right)$, since both $y$ and $L$ are known (see the drawing).

**SOLUTION** We will first compute the angle between the central bright fringe and the seventh dark fringe using the geometry shown in the drawing:

$$\theta = \tan^{-1} \left( \frac{y}{L} \right) = \tan^{-1} \left( \frac{0.025 \text{ m}}{1.1 \text{ m}} \right) = 1.3^\circ$$

The wavelength of the light is

$$\lambda = \frac{d \sin \theta}{m + \frac{1}{2}} = \frac{(1.4 \times 10^{-4} \text{ m}) \sin 1.3^\circ}{6 + \frac{1}{2}} = 4.9 \times 10^{-7} \text{ m}$$

10. Light of wavelength 691 nm (in vacuum) is incident perpendicularly on a soap film ($n=1.33$) suspended in air. What are the two smallest nonzero film thicknesses (in nm) for which the reflected light undergoes constructive interference?

**REASONING** When the light strikes the film from above, the wave reflected from the top surface of the film undergoes a phase shift that is equivalent to one-half of a wavelength, since the light travels from a smaller refractive index ($n_{\text{air}} = 1.00$) toward a larger refractive index ($n_{\text{film}} = 1.33$). On the other hand, there is no phase shift when the light reflects from the bottom surface of the film, since the light travels from a larger refractive index ($n_{\text{film}} = 1.33$) toward a smaller refractive index.
(\(n_{\text{air}} = 1.00\)). Thus, the net phase change due to reflection from the two surfaces is equivalent to one-half of a wavelength in the film. This half-wavelength must be combined with the extra distance \(2t\) traveled by the wave reflected from the bottom surface, where \(t\) is the film thickness. Thus, the condition for constructive interference is

\[
\frac{2t}{\lambda_{\text{film}}} + \frac{1}{2}\lambda_{\text{film}} = \lambda_{\text{film}}, 2\lambda_{\text{film}}, 3\lambda_{\text{film}}, \ldots
\]

We will use this relation to find the two smallest non-zero film thicknesses for which constructive interference occurs in the reflected light.

**SOLUTION** The smallest film thickness occurs when the condition for constructive interference is \(\lambda_{\text{film}}\). Then, the relation above becomes

\[
2t + \frac{1}{2}\lambda_{\text{film}} = \lambda_{\text{film}} \quad \text{or} \quad t = \frac{1}{4}\lambda_{\text{film}}
\]

Since \(\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}\) (Equation 27.3), we have that

\[
t = \frac{1}{4}\lambda_{\text{film}} = \frac{1}{4}\left(\frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}\right) = \frac{1}{4}\left(\frac{691 \text{ nm}}{1.33}\right) = 1.30 \times 10^2 \text{ nm}
\]

The next smallest film thickness occurs when the condition for constructive interference is \(2\lambda_{\text{film}}\). Then, we have

\[
2t + \frac{1}{2}\lambda_{\text{film}} = 2\lambda_{\text{film}} \quad \text{or} \quad t = \frac{3}{4}\lambda_{\text{film}} = \frac{3}{4}\left(\frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}\right) = \frac{3}{4}\left(\frac{691 \text{ nm}}{1.33}\right) = 3.90 \times 10^2 \text{ nm}
\]

20. A slit whose width is \(4.30 \times 10^{-5}\) m is located 1.32 m from a flat screen. Light shines through the slit and falls on the screen. Find the width of the central fringe of the diffraction pattern when the wavelength of the light is 635 nm.

**REASONING AND SOLUTION** The position of the first minimum is \(y = L \tan \theta\) (see Example 6), so the width of the central maximum is \(2y = 2L \tan \theta\). We know that \(\theta = \sin^{-1}\left(\frac{\lambda}{W}\right)\). For \(\lambda = 635 \text{ nm}\) and \(W = 4.30 \times 10^{-5}\) m, \(\theta = 0.846^\circ\) and

\[
2y = 2(1.32 \text{ m})(\tan 0.846^\circ) = 0.0390 \text{ m}
\]