Assignment:
Tuesday: Pr. 9.1, Chapter 10: Derive the central results as completely as possible – demonstrate a mastery of the material and techniques. Be prepared to lead class. I will call on you guys in turn to come up to the board and lead us through – that’s where your deep thoughts points will come from.

Administrative

- Exam Feedback
  - People’s scores pretty accurately reflected their level of mastery of the material, which in turn reflected how much effort they put into learning it. None of you should have been terribly surprised by the exam – it was very much like the practice exam, homework, and lectures. Similarly, none of you should be terribly surprised at how you did since it was right in line with how you were performing on the coursework. However, I believe that all of you have the potential to do better and I urge you to realize that potential – a big key to that is getting the homework done – apply yourselves, work together, come see me for help.

- Due dates
  - The people who did the worst on the test are the people who didn’t turn in their homework. It is my suspicion that they expected to eventually get around to doing the homework, but never did until it was too late. So, to look out for these folks, I’m going to have to return to firm due dates. If it isn’t in when it’s due, it isn’t graded.

- Where we’ve been: Perfect Classical Gas
  - Gas – Indistinguishable particles
  - Perfect – Non-interactive
  - Classical – we ignore Pauli Exclusion principle
  - Condition of Applicability – temperature is high enough that particles are, for the most part, sparsely distributed among states: \( n \ll 1 \). This could be re-stated as \( \tilde{l} \ll \lambda_r \), the average separation of particles in the gas is much greater than the quantum mechanical wavelengths of the particles. This condition is satisfied under many experimental conditions, thus the Perfect (Ideal) Gas Law applies to many gasses – but not to all. One counter example is a ‘gas’ of conduction electrons flowing around in a metal – their wavelengths are huge compared to their separations. Another counter example is light – more about that soon.

- Where we’re gong: Perfect Quantal Gas
  - Get it cool enough, and most any system will start displaying non-Classical behavior.
  - Perfect Quantal Gas Formalism: Today we will start all over again with gasses. This time, we will not make any of the approximations that we did last time; this time, we will directly address the Pauli Exclusion Principle.
    - Good: The upside is that we will have a formalism that is correct even for very low temperatures and exotic gasses.
- **Bad:** The downside is that, without knowing more about our specific system, we will not be able to get very far – no general expression for average energy, entropy, heat capacity…
  - **Perfect Quantal Gas Example:** To get some specific expressions, we’ll turn to a specific example – a gas of photons.
- **Black Body Radiation:** In the context of the historical development of quantum mechanics, most of you have already encountered this system and seen the basic results. Today we will see how its theoretical description is developed from statistical mechanics, and just where \( h \) first appeared.

### Chapter 9  Perfect Quantal Gas

- **Quantum Statistics of Truly Identical Particles:** Fermions and Bosons
  - **Motivation & Background:**
    - For reasons that I’ve never seen perfectly explained, there is a fundamental relation between a particle’s quantum mechanical spin and its statistics. I’ll pull together the easy parts of the story.
  - **Spin:** All fundamental particles have inherent angular momentum, even if they are not orbiting anything. This can be understood in terms of relativistic quantum field theory, but short of that, we often appeal to the classical analogy of a planet: even if it were not orbiting the sun, it would have angular momentum associated with its *spinning* on its axis. Like all other properties in quantum mechanics, the magnitude of particle ‘spin’ is quantized.
    - \( s_z = \frac{1}{2} h \) *Particles:* Most sub-atomic particles have spins of magnitude \( s_z = \frac{1}{2} h \) (note: \( h \) does have dimensions of angular momentum: distance times mass times velocity), these include electrons, protons, neutrons, quarks.
  - **Composite Particles:** As in classical mechanics, the angular momentum of a system of particles is the vector sum of their individual angular momenta. This holds for the spin of a collection of, say protons, neutrons, and electrons.
    - **Ex: Hydrogen:** Take a hydrogen atom, with one proton and one electron. Either they are both spinning the same way, say clockwise, or they are spinning in opposite ways. So \( s_H = s_e \pm s_p = \frac{1}{2} h \pm \frac{1}{2} h = h, 0 \).
      - **Note:** you have two spin-1/2 particles, and the system has an integer spin (1 or 0).
    - **Ex: Deuterium:** Some hydrogen atoms have a neutron along with the proton in the nucleus, this is known as Deuterium. Now the possible spin of the
whole system is
\[ s_D = s_e \pm s_p \pm s_n = \frac{1}{2} h \pm \frac{1}{2} h \pm \frac{1}{2} h = \frac{1}{2} h, \frac{3}{2} h \]

- **Note:** There are 3 spin-1/2 particles, and the composite particle has a \( \frac{1}{2} \)-integer spin.

- **Moral:** A collection of an odd number of spin-1/2 particles (electrons, protons, and neutrons) has a \( \frac{1}{2} \) integer spin; A collection of an even number of spin-1/2 particles has a full integer spin.
- **Fermion:** a particle with \( \frac{1}{2} \)-integer spin
- **Boson:** a particle with full-integer spin

- **Particles:** Light can be characterized as coming in nuggets, known as photons, having an inherent angular momentum of magnitude \( s = \hbar \). There are some exotic & short lived subatomic particles that are built of an even number of quarks; thus they have integer spin.

- **Composite Wave functions:** Say you have two similar particles to play with, particle A and particle B; perhaps they’re both electrons.
  - We put both particles in a box together. The state of a particle is completely described by its energy, its spin, the type of particle it is, and the fact that it’s confined to the box. For brevity, let’s just say that particle A is in state 1 and particle B is in state 2. A quantum mechanical wave function for this system could be found, \( \Phi(A(\mathrm{state}1), B(\mathrm{state}2)) \). Then the probability of the system being in such a state is, according to quantum mechanics,
    \[ P(A(1), B(2)) = |\Phi(A(\mathrm{state}1), B(\mathrm{state}2))|^2. \]
  - Then again, the two particles are of the same type, so if we swapped them, moved particle B down to state 1 and particle A up to state 2, that arrangement should be just as likely. It follows that:
    \[ P(A(2), B(1)) = P(A(1), B(2)) \]
    \[ |\Phi(A(\mathrm{state}2), B(\mathrm{state}1))|^2 = |\Phi(A(\mathrm{state}1), B(\mathrm{state}2))|^2 \]
    \[ \Phi(A(\mathrm{state}2), B(\mathrm{state}1)) = \pm \Phi(A(\mathrm{state}1), B(\mathrm{state}2)) \]
  - It turns out that, if we are talking about \( \frac{1}{2} \)-integer spin particles, fermions, it’s the – sign; if we are talking about full-integer spin particles, bosons, it’s the + sign. This is the point that I’ve not seen theoretically explained (though it may be in relativistic quantum field theory).
    - **Now, what if we put the two particles in the same state at the same time?**
      \[ \Phi(A(\mathrm{state}1), B(\mathrm{state}1)) = \pm \Phi(A(\mathrm{state}1), B(\mathrm{state}1)) \]
    - **Bosons:**
\( \Phi(A(\text{state}_1), B(\text{state}_1)) = \Phi(A(\text{state}_1), B(\text{state}_1)) \) 
the math gives us a trivial identity, something equals itself. Whoop-de-do.

- **Interpret**: Two bosons can be in the exact same state.

  - **Fermions**:
    \( \Phi(A(\text{state}_1), B(\text{state}_1)) = -\Phi(A(\text{state}_1), B(\text{state}_1)) \)
    whoops! Something is its own opposite, there’s only one something for which that is true: 0.

  - **Interpret**: This tells us that the wave function, and thus the probability of two fermions being in the same state is 0 – can’t be done.

  - **Pauli Exclusion Principle**
    - This is simply our last observation: no more than one Fermion can be in a particular state.

  - **Statistics**
    - Where we begin is the energy states of our system. Once we can define them, we can the write the partition function, and from that we can find the average energy, heat capacity, entropy, pressure…
    - The energy of the gas in a particular state depends on how many particles are in which particle state
      \[ E_{\text{gas, state}} = \sum_{\text{particle, state}} n_{\text{particle, state}} e_{\text{particle, state}} \]
      where, of course,
      \[ N = \sum_{\text{particle, state}} n_{\text{particle, state}} \]
      - For fermions, \( n_{\text{particle, state}} \) is 0, 1; either there is one or none in a given particle state.
      - For bosons, \( n_{\text{particle, state}} \) is 0, 1, 2, 3, ... any number can be in any particle state
      \[ Z_{\text{gas}} = \sum_{\text{gas, state}} e^{-\beta E_{\text{gas, state}}} = \sum_{\text{gas, state}} e^{-\beta \sum_{\text{particle, state}} n_{\text{particle, state}} k_{\text{particle, state}}} \]
      - Without knowing more about our particles, this is as far as we can go.

### Chapter 10: Black Body Radiation

- **Black-Body**: Any body that radiates thermally equilibrated light. That means that any incident light is absorbed and then re-emitted.
  - **Q**: How does light interact with matter?
  - **A**: Classically, we’d say that light is produced when a charged particle accelerates – kinking the electric and magnetic fields around it. Quantum mechanically, we’d say that light is produced when a charged particle changes energy levels –
shedding excess energy as light. Light is absorbed by the opposite process.

- **Qualitative: Thermally equilibrated light**
  - Focusing for a moment on the particles in a solid. What does the distribution of particle energies look like?
    - A: somewhat Gaussian, similar to the distribution of speeds in a gas.
  - The particles are constantly getting excited and de-excited – constantly absorbing and emitting light. The light emitted will have a similar distribution of energies to that of the particles – we saw a similar thing with the HCl spectrum in lab.
  - If the particle energies are broadly distributed, then we expect the energies of emitted light to be similarly distributed. Furthermore, the distribution of light energies should be tied to the material’s temperature: The average light energy should be a function of temperature.

- **Setting-up the problem**
  - How to approach the problem? Each time a vibration in the solid de-excites, light is emitted; we’ll call this nugget of light a Photon. It carries away energy equal to the change in the emitting particle’s energies, \( \epsilon_r \).
  - Now, similar de-excitations can happen all over in the material, producing any number of photons of the same energy.
    - Q: Treating these nuggets of light as particles, does that sound like a fermion or a boson?
      - A: boson. \( n_r = 0,1,2,\ldots \) for any \( r \).
      - However: \( N = \sum_r n_r \) needn’t hold. We can have any number of morsels of light emitted; the total number need not be conserved.