Monday  12.4-12.6 Thermal Expansion, Heat  HW26: Ch14; Pr. 30 34, 50
Wednesday 12.7-12.11 Specific Heat & Humidity HW25Redo; HW27: Ch12; Pr. 11, 21, 25, 30

Equipment

- **Solid thermal expansion**
  - Metal ring and ball
  - Blow torch
- **Different thermal expansion coefficients**
  - Bimetalic strip
  - Bimetallic strip circuit
  - Glass jar and metal lid
- **Thermal contraction**
  - A frozen, bulging water bottle – water/ice,
- **Liquid thermal expansion**
  - Thermometer
  - Beaker & hot plate

Last time:

- Kinetic Theory / Statistical Mechanics of an ensemble of individual particles
  - Diffusion

Statistical Distribution

- A cube of gas of identical particles (like Ar atoms)
  - \[ \text{Prob}(v) = P_e dv = \frac{1}{\sqrt{\pi} kT} \left( \frac{m}{kT} \right)^{1/2} v^2 e^{-\frac{m}{kT} v^2} dv \]

- Average Energy
  - \[ \langle KE \rangle = \frac{1}{2} kT \]
  - \[ PV = N \frac{2}{3} \left( \langle m \langle v^2 \rangle \rangle \right) \]
  - \[ (KE.) = \left( \frac{1}{2} m \langle v^2 \rangle \right) = \frac{1}{2} \frac{PV}{N} \]

This Time

Thermal Expansion

Intro / transition

Today, we’re going to use our atomistic picture of matter to understand the phenomenon of thermal expansion. This is a phenomenon that can do nasty things like mangle railroad tracks running through the desert, or helpful things like help maintain a pleasant temperature in this lecture hall. It is also a phenomenon with very basic, atomic scale origins.
**Question:** What happens to the volume of a gas in a balloon volume when it is heated? Why?

**A:** It expands. The hotter the gas is, the more kinetic energy the particles have, the harder they hit into the walls and thus the more they push the walls out. \(\frac{V_2}{V_1} = \frac{T_2}{T_1}\), all else held constant.

**Question:** Why doesn’t the balloon keep expanding indefinitely?

**A:** First, the expanded volume means expanded distance the particles travel between collisions with the wall, so the reduced time average number of collisions, so the pressure is kept in check. Second, the balloon is rubbery – the more you stretch it, the harder it is to further stretch it. So the chemical bonds holding the rubbery balloon walls together pull harder, to balance the harder push of the gas.

This is an example of *thermal expansion*.

- **Thermal Expansion – Interplay of kinetic and potential energy**

**Intro:** In general, temperature means particle motion. The hotter a collection of particles is, the more they move, whether that collection is a gas where the particles are negligibly attracted to each other and thus move freely, a liquid where they are weakly attracted and thus have a harder time moving about, or a solid in which they are strongly attracted to each other, and thus seldom travel far. The interplay between attractive forces pulling particles to each other and their random thermal motion – tending to separate particles, determine how closely packed the collection of particles is. Heat up the collection, be it gas, liquid, or solid, and the individual particles will separate out a little, cool it off, and they will come closer together. On the macroscopic scale, this means that the volume the collection of particles occupies grows larger when it is heated and shrinks when cooled.

We’ll try to picture this for our 3 states of mater, and then work a little quantitatively with this Thermal Expansion.

- **Gas:** Think about a gas, held in a balloon. The balloon walls are held out by the pressure of the molecules slamming into them. There is a balance between the tension in the balloon walls (and outside atmospheric pressure) and the force of the molecules slamming into the walls. Heat the balloon up and the molecules slam harder into the walls.

  - **Question:** What happens to the size of the balloon?
    - The balloon walls are pressed out further, the strain in the walls increase until a new balance is reached between the increased force of the molecules slamming into the walls, trying to escape, and the tension in the walls.

  - In very general terms, there is an interplay between the force of particles trying to escape, and some containing force, holding them together; the hotter a collection of particles gets, the more the particles want to fly apart – the bigger the collection gets.

- **Liquid:** There is a similar interplay within a liquid. In a liquid, there is a mild inter-molecular attraction which keeps the molecules relatively near each other (this replaces the tension in the balloon walls for a gas). However, just as with the gas, the molecules bounce around, and the hotter they get, the more violently they bounce, so they push each other out and
away. Their general attraction holds the fluid together, but a collection of molecules in a fluid will try to expand when heated.

- **Solid**: In a very general sense, the same is true for solids. Though the individual particles are not free to flow around because the inter-particle bonds are much stronger than in a fluid and quite much stronger than in a gas, the atoms can jiggle in place. The hotter they are, the more they jiggle, and it turns out that they spread-out just a little.

**Demo**: Pass ball through hoop.

**Question**: If I get the ball really hot, will it pass more easily, just as easily, or more difficulty / not at all?

Heat metal ball and try to pass through hoop.

What’s going on here?

- **Different Phases Expand Differently**: Going from Gas, to liquid, to solid the bonds that hold the particles to each other are greater and greater, correspondingly, the effect of heating each by the same amount is less and less expansion. You heat a can of hairspray, and it explodes. You heat a full glass of water and it overflows just a little, you heat a chunk of metal and it too grows, but very little.

- **Different Materials Expand Differently**: Each material varies in the strength, number, and orientation of its interatomic bonds, and these are what hold the material together in opposition to thermal motion.

**Question**: Should the expansion of the different materials, with their different bonds be different? Of course.

- Now say you have two different materials side by side, like the glass of a jar and the metal of its lid. If the fit is just a little too tight, what do you do?
  - Run hot water – the metal expands more than the glass, so the fit loosens.
- Or water in a metal hot water heater – My old house had radiators – hot water flowed through them, to accommodate for thermal expansion of the water, I had an overflow tank to catch the extra water that is expanded beyond the radiators’ capacity.

### 12.4 Linear Thermal Expansion

- **Where equation comes from**: Using quantum mechanics, one can model the bonding of atoms to each other, and combining that with statistical mechanics, one can find the average change in atomic separation that accompanies a change in temperature. Say neighboring atoms in a chunk of Al are initially exactly \(3 \times 10^{-10} \text{m}\) apart. You heat you chunk of Al by 1 K and it lengthens by \(6.9 \times 10^{-15} \text{m}\) or experiences a \(\frac{\Delta L}{L} = 23 \times 10^{-6}\) relative change for that 1° change in temperature.

Zooming out, focus on three of the atoms in a row. Each bond lengthens the same
amount, \( \frac{\Delta L}{L} = \frac{2 \cdot 6.9 \times 10^{-15}}{2 \cdot 3 \times 10^{-10}} = 23 \times 10^{-6} \); or say we look at a long rod of Al, the change in length scales right up with the length of the piece preserving the same relative change.

- Now say that the raise the temperature by 2\(^\circ\) instead of 1\(^\circ\), the relative change is twice as much. Writing this out as a general equation: \( \frac{\Delta L}{L_0} = \alpha_{Al} \Delta T \), where \( \alpha_{Al} = 23 \times 10^{-6} / \, ^\circ C \) is called the Coefficient of Linear Expansion for Aluminum.

- This relationship holds generally for materials other than Al; however, each different material has different strength bonds, different atomic masses, etc. so their relative growths differ, i.e., they have different coefficient’s of linear expansion.

  - Note: That the expansion increases linearly with the temperature is not simply a happy coincidence. The scales of temperature were originally defined by this very phenomena. Heat Hg and it climbs up a hollow column in a glass tube. This is linear expansion. Call the height the Hg rises to at waters boiling point 100\(^\circ\), call its height at water’s freezing point 0\(^\circ\). Divide the length in between in to equal intervals. This defines the °C in terms of a linear distance.

The different coefficients are found in Table 12.1. Again, since each material has bonds of different strength, length, and number; each material expands differently, so it has a different coefficient.

### 12.4.1.1 Equation 12.2: Linear Thermal Expansion of A Solid

- \( \frac{\Delta L}{L_0} = \alpha \Delta T \)
- \( \alpha = \) Coefficient of Linear Expansion
  - Different for different material
  - Units = 1/ °C = 1/ °K

### Simple Example 1, Like 14, Pr 13

A steel beam is used in the construction of a skyscraper. What is it’s relative growth from the dead of winter (-30 °F) to the height of summer (100 °F)?

- \( \frac{\Delta L}{L_0} = \alpha_{steel} \Delta T \), \( a_{steel} = 12 \times 10^{-6} / ^\circ C \)
- \( \Delta T = 110 \, ^\circ F - 30 \, ^\circ F = 43.3 \, ^\circ C + 34.4 \, ^\circ C = 77.8 \, ^\circ C \)
- \( \frac{\Delta L}{L_0} = 12 \times 10^{-6} \times 77.8 \, ^\circ C = 9.3 \times 10^{-4} \)

CQ 2 Back when the international standard of length was defined as the separation of two fine marking on a metal bar, why was it necessary to maintain that bar at a constant temperature?
- So it wouldn’t grow or shrink.
Concrete and steel happen to have very similar constants of thermal expansion. Why is this important when steel is used to reinforce concrete highway structures?

- It means that the relative growth of the two materials doesn’t cause them to separate from each other or to break themselves – they grow and shrink together, as a unit.

There is a range of applications of thermal expansion – some good, bimetallic strips regulating temperature in household appliances, hot water helping to open jars of spaghetti sauce, railroad rails destroyed under the desert sun.

12.4.2 Bimetallic strip

- Say you take strips of two materials with different coefficients of thermal expansion and weld them together like two slices of bread. Then you heat them. What will happen?

- **Demo:** Heat bimetallic strip
  - Each one will try to expand according to \( \frac{\Delta L}{L_0} = \alpha \Delta T \). The one with the greatest Coefficient of Linear Thermal Expansion will try to expand more than the other. If the strips are thin and bendable, the two strips can each do (mostly) what they want to. By bending, the one that expands more goes on the outside, the one that expands less goes on the inside of the curve; comparing radii of curvature, the outer one has a slightly greater radius, thus it a large arc length.
  - Use: a lot of small household appliances that are temperature sensitive use bimetallic to turn off when the appropriate temperature is achieved: thermostats, toasters, coffee makers.

12.4.3 Thermal Stress

- That was a case where two materials with different coefficients of expansion were able to work out a compromise. Now, think about what happens when no compromise can be reached – one material is enclosed in or bolted down to another. Say a gas is heated in a vessel, it tries to expand but the vessel won’t let it. The collisions with the container walls become more violent / more forceful. This is seen in a raise in the pressure. The relationship between that raise in pressure and the raise in temperature is seen in the ideal gas law:
  - \( PV = kNT \), so, \( \Delta P = (kN/V) \Delta T \)

- The same phenomena occurs for solid chunks of material that are heated, yet confined, say a sheet of metal bolted down at its ends. The metal is heated, it tries to expand, but is confined, so strong internal pressures, stresses, develop due to the force of confinement.

12.4.3.1 Equation 10.17 b

- \( \Delta \text{Stress} = (Y\alpha) \Delta T \) (not fully given in book)
  - The book argues out this equation based on the equation for thermal expansion and an equation relating stress and externally forced expansion.
  - \( \alpha \) is the same coefficient of linear thermal expansion
  - \( Y \) is known as Young’s modulus and is a material dependent constant relating the internal stresses to expansion, found in table 10.1.
    - Like \( \alpha \), this can be derived from fundamental physical principles.
    - Units of pressure, N/m^2.
In lab the other day, I took a beaker off a hot plate and momentarily set it on the table top. One of you quickly & rightly cautioned me that that was risky. Why?
You don’t need two different materials to develop thermal stresses. These stresses develop in a material that is unevenly heated, part of it wants to expand more than other parts will let it.

**Example:** Invar is an alloy that has a coefficient of thermal expansion very close to that of glass. This allows laboratory instruments to be created with glass and metal mated directly to each other. Heat or cool them & they won’t break.

CQ 9 For baking (or Chemistry) Pyrex is often used rather than regular glass. With regards to thermal stress and coefficients of thermal expansion, why is Pyrex preferable?

- Pyrex has a smaller coefficient of thermal linear expansion and a smaller one of thermal volumetric expansion than does regular glass, presumably regular glass’s Young’s modulus does not offset this improvement, therefore, Pyrex is less stressed when its temperature changes locally.

**Example 2 Like 16:** Say the manufacturers of some Pyrex ware want to test the limits of their new an improved formula, with a coefficient of linear expansion of $3.2 \times 10^{-6} \, ^\circ C$. They rigidly mount a rod of it and measure the pressure it exerts on a sensor at on end when heated by 100 °C in order to determine its Young’s Modulus. Say it applies a pressure (experiences a stress) of $2.0 \times 10^{7} \, N/m^{2}$. What is the new material’s Young’s Modulus?

- $\Delta \text{Stress} = (Y \alpha) \Delta T$
- $Y = \frac{\Delta \text{Stress}}{\alpha \Delta T} = \frac{2 \times 10^{7} \, N/m^{2}}{3.2 \times 10^{-6} \, \nu_{c} \cdot 100 \, ^\circ C} = 6.25 \times 10^{10} \, N/m^{2}$

2-D Thermal Expansion.

- **Intro.** So far, we’ve only looked at one dimension of a chunk’s thermal expansion. But if length grows, for the same reason, and in the exact same way, width and depth should grow.
  - **Question:** What was the atomic scale reason I gave for the 1-D expansion?

- Let’s pause on the way between 1-D and 3-D growth, and look at 2-D.

- **Motivational Example / Derivation.** Say you have a sheet of glass, perhaps for a window. Say on a cool morning it measures 0.25 m by 0.5 m, to have an area of $A = L \times W = 0.125 \, m^{2}$. By how much have the dimensions changed when measured at mid-day when it’s 6 °C warmer? What is the relative change in area?

  - $\frac{\Delta L}{L_{0}} = \alpha_{\text{glass}} \Delta T$, $\frac{\Delta W}{W_{0}} = \alpha_{\text{glass}} \Delta T$
  - $\Delta L = L_{0} \alpha_{\text{glass}} \Delta T = 0.25 \, m \cdot 8.5 \times 10^{-6} \, \nu_{c} \cdot 6 \, ^\circ C = 1.3 \times 10^{-5} \, m$
  - $\Delta W = W_{0} \alpha_{\text{glass}} \Delta T = 0.5 \, m \cdot 8.5 \times 10^{-6} \, \nu_{c} \cdot 6 \, ^\circ C = 2.6 \times 10^{-5} \, m$
\[
\frac{\Delta A}{A} = \frac{A - A_0}{A_0} = (L_0 + \Delta L) \times (W_0 + \Delta W) - A_0 = (L_0 + L_0\alpha \Delta T) \times (W_0 + W_0\alpha \Delta T) - A_0
\]

\[
\frac{\Delta A}{A} = \frac{L_0 \times W_0 + L_0 \times W_0\alpha \Delta T + L_0\alpha \Delta T \times W_0 + L_0 \times W_0 \alpha \Delta T}{A_0} - A_0
\]

\[
\frac{\Delta A}{A_0} = A_0 + A_0\alpha \Delta T + A_0\alpha \Delta T + A_0(\alpha \Delta T)^2 = A_0 \alpha \Delta T + A_0(\alpha \Delta T)^2
\]

So in this example, \( \frac{\Delta A}{A} \approx (2\alpha) \Delta T = 2 \cdot 8.5 \times 10^{-6} \\gamma_c - 6^\circ C = \frac{1.02 \times 10^{-5}}{2} \)

- The 2\(a\) then acts as the Area Thermal Expansion Coefficient.

12.4.4 Expansion of Holes

- Say you have a chunk of material, with a hole cut out of it. When heated the material expands. Does the hole get bigger or smaller?
- **Demo:** Heat ball and not be able to pass through ring. Heat ring too, and be able to pass through just like neither were heated.
  - Rather counter intuitively, the hole gets bigger.
  - Let’s see why.
  - The book chooses a nice simple example of an arrangement of 8 tiles:

  - **Initially at** \(T_i\)

Consider one tile first, originally measuring \(L_0 \times W_0\). Looking at our arrangement of tiles, this means that the whole also measures \(L_0 \times W_0\).
Finally at $T_f$

It’s length changes according to $\frac{\Delta L}{L_0} = \alpha \Delta T$, the relative change in its width follows the same rule: $\frac{\Delta W}{W_0} = \alpha \Delta T$.

So its dimensions increase to $W_f = W_0 + \Delta W$ by $L_f = L_0 + \Delta L$. Similarly for the hole.

**The bottom line** is that the hole’s expansion is just the same as a solid chunk of the material which surrounds it.

**Explanation:** With the tiles this can be seen because when the tiles above and below expand, they push the tiles to the left and right further out, in spite of the fact that the tiles to the left and right are expanding themselves. If we removed the top and bottom tiles, then the left and right tiles would be free to grow into the gap. Similarly the tiles on the left and right push the top and bottom tiles out, making the gap between them grow.

### 12.5 Volume Thermal Expansion

1. Now say instead of a mostly 1-D rod, or 2-D sheet, we’ve got a full 3-D object. The thermal expansion of each of its dimensions (length, width, and height) results in an overall expansion in its volume $V = L \times W \times H$.

2. As we derived the expression for relative area expansion, we could do the same for relative volume expansion. Here’s the set-up, the rest can be done for a bonus HW problem.

   - $\frac{\Delta V}{V_0} = \frac{V - V_0}{V_0} = \frac{(L_0 + \Delta L)(W_0 + \Delta W)(H_0 + \Delta H) - V_0}{V_0} = \ldots$

   - By comparing the linear expansion and area expansion expressions, you can guess what the volume expansion expression will be.

   - $\frac{\Delta L}{L_0} = \alpha \Delta T$, $\frac{\Delta A}{A_0} \approx (2\alpha) \Delta T$, $\frac{\Delta V}{V_0} \approx (3\alpha) \Delta T = \beta \Delta T$

### 12.5.1 Equation 12.3

- $\frac{\Delta V}{V_0} = \beta \Delta T$,  

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**Note:** The diagram shows a 3x3 grid of tiles with one tile highlighted as a hole. The dimensions and changes are marked as $L_0$, $L_f$, $W_0$, $W_f$, and $V_0$, $V_f$.
Coefficient of Volume Expansion: For convenience, the $3\alpha$ is named $\beta$, and table 12.1 gives the $\beta$ values for different materials, but you’ll observe that each value is equal to 3 times the $\alpha$.

12.5.2 ClassWork: By how much would you have to heat a chunk of aluminum to make it grow by 1%?

CQ 11 How do the buoyancy forces on an object differ in cold vs. hot water?

- You have greater buoyancy in cold water. The reason is that the water compresses when it gets cold – so the same amount occupies a smaller volume, i.e., it has a lower density. $F_B = w_{\text{water}} = \rho_w V_{\text{disp}} g$

Example 4 Table 11.1 says that at 4°C the density of water is $1 \times 10^3$ kg/m$^3$. How many times more dense is water at 4°C than at 80 °F = 27 °C?

- $\rho_{4C} = \frac{M}{V_{4C}}$, $\rho_{27C} = \frac{M}{V_{27C}}$, $V_{27C} = V_{4C} + \Delta V$, $\frac{\Delta V}{V_0} \approx \beta \Delta T$

- $\frac{\rho_{27C}}{\rho_{4C}} = \frac{V_{4C} + |\Delta V|}{V_{4C}} = 1 + \beta \Delta T = 1 + 207 \times 10^{-6} \gamma_C (27^\circ C - 4^\circ C) = 1.005$

Example 5 combining Volume Expansion and Hole Expansion Like 25

- Gasoline: You know you’re not supposed to leave a full can of gas in the sun. Let’s see what can happen. Say we have a 1 gallon, or $3.8 \times 10^{-3}$ m$^3$ of gasoline filling an Al gas can early in the morning out in the dessert – say 10°C, in the heat of the day, 43 °C,

  - A) What volume would the gas like to occupy?

    - $V_{\text{gas}} = V_0 + \Delta V_{\text{gas}}$, $\frac{\Delta V}{V_0} \approx \beta \Delta T$

    - $V_{\text{gas}} = V_0 + V_0 \beta_{\text{gas}} \Delta T = V_0 (1 + \beta_{\text{gas}} \Delta T) = 3.8 \times 10^{-3} m^3 (1 + 950 \times 10^{-6} \gamma_C 33^\circ C)$

    - $V_{\text{gas}} = 3.919 \times 10^{-3} m^3$

  - B) What volume would does the can grow to?

    - We found in 2-D that a hole in a material expands just like a chunk of that material, the same holds for 3-D.

    - $V_{\text{can}} = V_0 + \Delta V_{\text{can}}$, $\frac{\Delta V}{V_0} \approx \beta \Delta T$

    - $V_{\text{can}} = V_0 + V_0 \beta_{\text{Al}} \Delta T = V_0 (1 + \beta_{\text{Al}} \Delta T) = 3.8 \times 10^{-3} m^3 (1 + 23 \times 10^{-6} \gamma_C 33^\circ C)$

    - $V_{\text{can}} = 3.803 \times 10^{-3} m^3$
C) How much gas spills out of the can?

\[ V_{\text{spill}} = V_{\text{gas}} - V_{\text{can}} = 3.919 \times 10^{-3} \text{ m}^3 - 3.803 \times 10^{-3} \text{ m}^3 = 1.2 \times 10^{-4} \text{ m}^3 = 0.03 \text{ gal} \]

Water

Demo: Bring a frozen, bulging water bottle & ask what happened.

- **Intro.** Because water is a substance with which we have everyday experience, and because it behaves a little differently than other materials, it is worth looking at more closely.

- **In general, materials compress as they cool.** This holds within and across phases: a cool gas will occupy less volume than would a hot gas (for same pressure); a cool liquid will occupy less volume than a hot liquid and a hot liquid occupies less volume than a cool gas, etc. The two competing elements are the thermal motion that tends to move particles around and away from each other and the inter-particle attraction that pulls them together. Ultimately, the attraction would cause particles to bond in a nice crystalline pattern.

- **Water is the exception.** It turns out that the crystal pattern has the molecules a little more spread out in ice than slightly warmer water molecules are in liquid phase. This means that as liquid water cools, it shrinks (like anything else) until 4°C, but as it approaches freezing, and the molecules tend toward their crystalline positions, water begins expanding again until it is cooled to freezing (4°C – 0°C).

- **Implications.**
  - **Thermal Strain:** This means that freezing a bottle of water or Pop may break it. It means that water frozen in a pipe can itself burst the pipe or can press enough on the remaining warm water that it bursts the pipe.
  - **Lake Freezing:** Water that is cooled by low air temperature becomes more dense and sinks, cycling the warmer, less dense water up to the surface. Since the densest water can be is when its 4°C, the cycle continues until the whole lake is 4°C. Then the surface water is cooled to 4°C and it stays at the surface. This water is further cooled and it begins to decompress, so it becomes less dense and is ensured to stay at the lake surface. Thus a frozen layer forms at the lake surface, rather than at the lake bottom (as would happen for a lake of most any other liquid freezing).
Ex. 1: A steel beam is used in the construction of a skyscraper. What is it’s relative growth from the dead of winter (-30 °F) to the height of summer (110 °F)?

Ex. 2: Say the manufacturers of some Pyrex ware want to test the limits of their new an improved formula, with a coefficient of linear expansion of $3.2 \times 10^{-6}/°C$. They rigidly mount a rod of it and measure the pressure it exerts on a sensor at one end when heated by 100 °C in order to determine its Young’s Modulus. Say it applies a pressure (experiences a stress) of $2.0 \times 10^7 N/m^2$. What is the new material’s Young’s Modulus?

Ex. 3: By how much would you have to heat a chunk of aluminum to make it grow by 1%?
Example 4: Table 11.1 says that at 4°C the density of water is $1 \times 10^3$ kg/m$^3$. How many times more dense is water at 4°C than at 80 °F = 27 °C?

Ex. 5: Gasoline: You know you’re not supposed to leave a full can of gas in the sun. Let’s see what can happen. Say we have a 1 gallon, or $3.8 \times 10^{-3}$ m$^3$ of gasoline filling an Al gas can early in the morning out in the dessert – say 10°C, in the heat of the day, 43 °C,

A) What volume would the gas like to occupy?

B) What volume does the can grow to?

C) How much gas spills out of the can?